UCLA Economics Dept

Econometrics Field Exam

FALL 2000

Please answer four of the seven questions. Use a separate blue book for each question.

1. **General Question:** Suppose that $X_1, X_2, \ldots$ are i.i.d. scalar random variables with (unknown) distribution function F. We are interested in estimating

$$\alpha \equiv Pr(X_i \leq 0)$$

based on a sample of size n. Consider the following estimator:

$$\hat{\alpha} = \frac{1}{n}\sum_{i=1}^{n} 1(X_i \leq 0).$$

(a) Prove that $\hat{\alpha}$ is unbiased.

(b) Prove that $\hat{\alpha}$ is consistent as $n \to \infty$.

(c) Derive the asymptotic distribution, as $n \to \infty$, of

$$\sqrt{n}(\hat{\alpha} - \alpha).$$

Try to give as complete a characterization of this asymptotic distribution as possible.

2. **General Question:**

The true conditional expectation function is given by

$$E(y_t|X,Z) = \beta_0 + \beta_1 x_t + \beta_2 z_t.$$

Assume that

$$V(y_t|X,Z) = \sigma^2, \quad \forall t.$$

Here X, and Z denote vectors of T observations on x and z respectively. Assume that you are dealing with time series data, and that x_t and z_t may be correlated. Unfortunately you omit z_t in your regression specification.

a. Determine the effect of omitting z_t on the sampling properties of the estimator for β_1.

b. What effects does the omission of z_t have on score tests for residual serial correlation? Use the construction of the test as well as the specification of z_t above in answering this question.

c. Suppose that you reject the null hypothesis for this residual serial correlation test. What model is specified under the alternative? What is the effect of using this alternative model on the sampling properties of the resulting estimator of β_1?

d. What effects does the omission of z_t have on score tests for heteroskedasticity, both with respect z_t and with respect to x_t? Use the construction of the tests as well as the specification of z_t above in answering this question.

e. Suppose that you reject the null hypothesis for these heteroskedasticity tests. What model is specified under the alternative? What is the effect of using this alternative model on the sampling properties of the resulting estimator of β_1?

f. Finally discuss the role of bootstrapping in assessing the sampling properties of the β_1 estimator in this short regression (ie. the regression with z_t omitted).

3. **231B:**

You want to fit an AR(p) model to some data.

a. Suppose you determine that $p = 2$. How will you estimate this model? How will you estimate the autocovariances of this model? Provide a mapping from the autoregressive parameters to the autocovariances. Specifically show how you would estimate the first four autocovariances.

b. What residual based procedure is relevant to assessing lag length? Describe this procedure. State the null and alternative hypotheses of this residual based test. Suppose you find evidence against the null hypothesis? How might you respond *besides* altering the lag length of your model?

c. How are the autocovariances and the moving average representation of this model related? Be precise in relating these two representations. How does the information in the autocovariances differ from that in the moving average representation? What features of the model are ignored by both the autocovariance function and the moving average specification? How do you compute the moving average terms?

d. You wish to correct for potential bias in your estimators for ρ_i in the specification:

$$(1 - \rho_1 L)(1 - \rho_2 L)y_t = \alpha + \varepsilon_t.$$

Above, L refers to the lag operator: $Ly_t = y_{t-1}$. What procedures would allow you to detect and correct for these biases? How would you make these corrections? How do your results in part (b) above affect your procedures here?

4. **232E:**

Consider a five variable VAR system. Suppose that $y_t = (y_{1t}, y_{2t}, y_{3t}, y_{4t}, y_{5t})'$, and that

$$y_t = a + \sum_{t \geq 0} C_j \varepsilon_{t-j},$$

with $E(\varepsilon_t \varepsilon_t') = \Sigma_\varepsilon$ diagonal. C_0 is of the form:

$$C_0 = \begin{bmatrix} X & X & X & X & X \\ 0 & X & 0 & 0 & X \\ 0 & 0 & X & X & 0 \\ 0 & 0 & X & X & 0 \\ 0 & X & 0 & 0 & X \end{bmatrix}$$

Here the X's indicate non-zero elements which are in general distinct from one another.

a. Specify which variables are predetermined in each equation.

b. Determine which variables can function as instruments in each equation.

c. Determine which equations are identified. Which equations are overidentified? Which are underidentified? Where possible evaluate the appropriate rank condition.

d. Let $C(L) = \sum_{j \geq 0} C_j L^j$, where L is the lag operator. In addition to the restrictions above, the long run matrix $C(1)$ has the same pattern of zero restrictions as C_0. What additional instruments does this generate? How do these extra restrictions affect identification?

5. We are interested in the effect of some binary treatment t_i on an outcome y_i. Consider the following model:

$$t_i^* = z_i'\gamma + x_i'\delta + \eta_i$$
$$y_{i0} = x_i'\beta_0 + \epsilon_{i0}$$
$$y_{i1} = x_i'\beta_1 + \epsilon_{i1}$$

Here x_i and z_i are vectors of exogenous variables, and x_i contains a constant. Assume that the treatment status is determined by

$$t_i = \begin{cases} 1 & \text{if } t_i^* > 0 \\ 0 & \text{otherwise} \end{cases}$$

and that the outcome is given by

$$y_i = t_i \cdot y_{i1} + (1 - t_i) \cdot y_{i0}.$$

In other words, the outcome is y_{i0} if $t_i = 0$ and y_{i1} if $t_i = 1$. Assume that

$$\begin{pmatrix} \eta_i \\ \epsilon_{i0} \\ \epsilon_{i1} \end{pmatrix} |x_i, z_i \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, \Sigma),$$

where

$$\Sigma = \begin{pmatrix} 1 & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{pmatrix}.$$

(a) Explain why it is reasonable to have the upper left element of Σ equal to 1.

(b) Assume that Σ is known and that the prior for $(\gamma, \delta, \beta_0, \beta_1)$ is "flat." You have n observations on (x_i, z_i, y_i, t_i). Outline a Gibbs sampling algorithm to simulate the posterior distribution of the parameters.

(c) Based on the Gibbs sampler in part (b), explain how to simulate

$$\tau \equiv \frac{1}{n}\sum_{i=1}^{n}(y_{i1} - y_{i0}).$$

What is the interpretation of τ?

6. Suppose the distribution of unemployment spells Y conditional on individual characteristics $X = x$ is exponential with parameter $\exp(x'\beta)$. You have a random sample of size N from this distribution.

(a) Suppose you only observe unemployment durations if they exceed 10 weeks. That is, if $Y > 10$, you observed both X and Y. If $Y < 10$ you do not observe either Y or X, and in fact you do not know that such a duration occurred. Write down the likelihood function.

(b) Consider the same sampling scheme as in (a), but now assume you also observe an indicator D for the event that $Y > 10$. Thus you observe $(D, D \times Y, D \times X)$. Write down the new likelihood function.

(c) Finally assume you also observe X if the duration is less than 10, but you only observe Y if $Y > 10$. Thus you observe $(D, D \times Y, X)$. Write down the likelihood function.

(d) From now on suppose you observe Y and X, irrespective of the value of Y. Calculate the large sample variance of the maximum likelihood estimator.

(e) What is the median of Y given X as a function of β?

(f) Describe how you could estimate β by doing median (quantile) regression.

(g) Calculate the asymptotic variance of the median regression estimator for Y. How does it compare to the asymptotic variance of the maximum likelihood estimator? Which should be bigger?

7. Suppose that

$$y_{it} = \phi y_{it-1} + \alpha_i + \varepsilon_{it} \qquad i = 1, ..., N; t = 1, 2, 3$$

Assume that the following assumptions hold:

(i) ε_{it} is uncorrelated with all lags of y_{it} and with α_i.

(ii) ε_{it} is homoskedastic over time.

(iii) $E(\varepsilon_{it}) = E(\alpha_i) = 0$ for all t.

(iv) The correlation between y_{it} and α_i is constant over time.

(v) y_{it} is homoskedastic for all t.

(a) Consider the following two estimators of ϕ in the model above: the first-difference estimator and the OLS estimator that ignores the presence of the individual effect. Are they consistent? Derive their probability limits.

(b) Describe in detail how you would estimate ϕ consistently and efficiently in the model above.