UCLA Economics

Econometrics Field Exam

Spring 2000

Please answer four of the seven questions. Use a separate blue book for each question.

General Questions:

1. (a) Suppose that you have 100 independent and identically distributed observations from a Poisson distribution with probability function

$$Pr(Y = y) = \frac{\lambda^y \exp(-\lambda)}{y!},$$

and suppose that their sample mean in 25. Construct the Lagrange Multiplier, Likelihood Ratio and Wald tests of the hypothesis that the mean of the underlying distribution equals 16.

(b) Suppose that conditional on $X = x$, Y has a Poisson distribution with mean $\exp(x'\beta)$. Is the non-linear least squares estimator based on minimizing

$$\sum_{i=1}^{N}(Y_i - \exp(X_i\beta))^2,$$

consistent for β_1? Is it efficient? Propose an alternative estimator and discuss its properties.

2. Consider the following demand and supply model:

$$q_i^d(p) = \alpha_0 + \alpha_1 \cdot p + \alpha_2 \cdot z_i + \varepsilon_i,$$

$$q_i^s(p) = \beta_0 + \beta_1 \cdot p + \beta_2 \cdot v_i + \eta_i.$$

The equilibrium price is the price that clears the market. It is assumed that it is unique in each market. The traded quantity is the quantity demanded at the equilibrium price. You can assume that ε and η are jointly independent of z and v. Let σ_ε^2 and σ_η^2 be the variances of ε and η, and let ρ be their correlation.

(a) Predict (in the sense of the Mean Squared Error criterion) the equilibrium price and quantity given $z = z_0$ and $v = v_0$.

(b) Predict (in the sense of the Mean Squared Error criterion) the quantity traded given that $z = z_0$, $v = v_0$ and the equilibrium price is p_e.

(c) Describe how you can estimate the coefficients in the demand equation from the moments of (z, v, p, q).

(d) Describe how you can test exogeneity of the price in the demand equation.

231A:

3. Suppose the distribution of Y conditional on $X = x$ is exponential with parameter $\exp(x'\beta)$. You have a random sample of size N from this distribution.

(a) What is the log likelihood function. How would it change if there is right censoring at a fixed time C?

(b) How would you calculate the maximum likelihood estimator?

(c) If $\hat{\beta}$ is the mle, how would you approximate its variance?

(d) Calculate the mean and variance of Y given $X = x$.

(e) Let $h(x, \beta)$ be the conditional expectation of Y given $X = x$. An alternative estimator for β is the gmm estimator based on the moment function $\psi(y, x, \beta) = x \cdot (y - h(x, \beta))$. Calculate the large sample variance of the gmm estimator based on this moment. How does it compare to the variance of the mle?

(f) What would you be testing by comparing the two estimators, $\hat{\beta}_{mle}$ and $\hat{\beta}_{gmm}$? What is the variance of the difference under the null that the model is exponential? You can assume that x is a scalar for the purposes of this question.

(g) Again assume x is a scalar. Show what the information matrix test would look like in this case.

Note on Exponential distribution: Recall that if Z has an exponential distribution with parameter θ, its probability density function is

$$f(z|\theta) = \theta e^{-\theta z} \qquad \text{for } z > 0.$$

The mean and variance of the exponential distribution are:

$$E(Z) = \frac{1}{\theta}, \qquad Var(Z) = \frac{1}{\theta^2}.$$

231B:

4. You want to fit an AR(p) model to some data.

(a) How do you determine the lag length p? Are certain methods more appropriate if your objective is to use this model for forecasting?

(b) Provide two tests of nonstationarity for this model. The tests should reverse the null and alternative hypotheses. Be explicit as to which test maintains which null hypothesis. Be explicit as to how to conduct each test. What is the motivation for considering these two tests together in assessing nonstationarity?

(c) Describe a prior you would use to conduct Bayesian inference for this model. Your prior should:

1. capture or emphasize the potentially nonstationary specification of the AR(p) model;
2. deal with uncertainty regarding the appropriate lag length;
3. treat both the autoregressive parameters, and the variance parameter as random variables.

Be explicit as to how you could incorporate these features into your prior. Describe the inference procedures that are appropriate to the prior you have chosen.

(d) Could you use this prior to conduct a Bayesian test for nonstationarity? Justify your answer. Contrast this prior with the one used by Schotman & Van Dijk.

(e) Suppose you constructed the test statistic

$$\hat{w}_4 = \frac{\sum_t \hat{\varepsilon}_t^4 / T}{\hat{\sigma}^4} - 3.$$

Here $\hat{\varepsilon}_t$ is the residual from the classically fitted AR(p) model above. What could this statistic be used to test? Specify a null and alternative hypothesis.

(f) You wish to use simulation techniques in estimating your model. Contrast Monte Carlo methods with bootstrap procedures. Motivate these procedures by appealing to previous parts of this question. That is, provide scenarios of diagnostic test outcomes where monte carlo methods might be supported, and alternative scenarios of diagnostic test outcomes where various bootstrap procedures might be preferred.

232D:

5. (a) Suppose that

$$y_{it} = 1\left\{x_{it}\beta + \alpha_i - \varepsilon_{it} \geq 0\right\} \qquad i = 1, \ldots, N; t = 1, 2$$

where ε_{it} are i.i.d. unobservable variables that are distributed logistic over time, x_{it} is a $1 \times k$ vector of observable variables, and α_i is an unobservable individual-specific effect. This implies that

$$\Pr\left(y_{it} = 1 | x_i, \alpha_i\right) = \frac{\exp\left(x_{it}\beta + \alpha_i\right)}{1 + \exp\left(x_{it}\beta + \alpha_i\right)}$$

where $x_i = \left(x_{i1}, x_{i2}\right)$. Discuss estimation of β. Assume that cross-sectional sampling is random.

(b) Now suppose that

$$y_{it} = 1\left\{\phi y_{it-1} + \alpha_i - \varepsilon_{it} \geq 0\right\} \qquad i = 1, \ldots, N; t = 1, 2, 3$$

where ε_{it} are i.i.d. logistic over time and α_i is an individual-specific effect. This implies that

$$\Pr\left(y_{it} = 1 | y_{i0}, \ldots, y_{it-1}, \alpha_i\right) = \frac{\exp\left(\phi y_{it-1} + \alpha_i\right)}{1 + \exp\left(\phi y_{it-1} + \alpha_i\right)}$$

Assume that cross-sectional sampling is random and that y_{i0} is observed for all individuals with

$$\Pr\left(y_{i0} = 1 | \alpha_i\right) = p_0\left(\alpha_i\right)$$

where p_0 is an unknown function. Let A denote the event

$$A = \left\{y_{i0} = d_0, y_{i1} = 0, y_{i2} = 1, y_{i3} = d_3\right\}$$

and B denote its complement,

$$B = \left\{y_{i0} = d_0, y_{i1} = 1, y_{i2} = 0, y_{i3} = d_3\right\}$$

(i) Derive the following conditional probabilities:

$$\Pr\left(A | A \cup B, \alpha_i\right) \qquad \text{and} \qquad \Pr\left(B | A \cup B, \alpha_i\right)$$

(ii) Based on (i) describe an estimator of ϕ.

232E:

6. Consider a five variable VAR system. Suppose that $y_t = (y_{1t}, y_{2t}, y_{3t}, y_{4t}, y_{5t})'$, and that

$$y_t = a + \sum_{t \geq 0} C_j \varepsilon_{t-j},$$

with $E(\varepsilon_t \varepsilon_t') = \Sigma_\varepsilon$ diagonal. C_0 is of the form:

$$C_0 = \begin{bmatrix} X & X & X & X & X \\ 0 & X & 0 & X & 0 \\ 0 & 0 & X & 0 & X \\ 0 & X & 0 & X & 0 \\ 0 & 0 & X & 0 & X \end{bmatrix}$$

Here the X's indicate non-zero elements which are in general distinct from one another.

(a) Specify which variables are predetermined in each equation.

(b) Determine which variables can function as instruments in each equation.

(c) Determine which equations are identified. Which equations are overidentified? Which are underidentified? Where possible evaluate the appropriate rank condition.

(d) How could you measure the relevance of instrumental variables? Be as specific as possible.

(e) Let $C(L) = \sum_{j \geq 0} C_j L^j$, where L is the lag operator. In addition to the restrictions above, the long run matrix $C(1)$ has the same pattern of zero restrictions as C_0. What additional instruments does this generate? How do these extra restrictions affect identification?

232F:

7. Suppose that for units $i = 1, \ldots, n$, we observe (Y_i, X_i). Y_i is a binary variable, and X_i is a continuous scalar. We assume that (Y_i, X_i) are jointly i.i.d. with some distribution, and we are interested in the conditional distribution of Y_i given X_i.

Let $g(x)$ denote the probability that $Y_i = 1$ conditional on $X_i = x$:

$$Pr(Y_i = 1|X_i = x) = g(x).$$

g takes on values in $[0, 1]$, and is assumed to be continuous and differentiable, but otherwise we know nothing about it. It is proposed to use a probit model, where the index is a polynomial in x:

$$Pr(Y_i = 1|X_i = x) = \Phi(\beta_0 + \beta_1 x + \beta_2 x^2 + \cdots + \beta_k x^k).$$

(a) Explain why this might be a reasonable approximate model to use even if it was not "correct" in the sense of being equal to the true $g(\cdot)$ for some choice of the parameters. (Hint: recall that any continuous function can be approximated by a polynomial with enough terms.)

(b) Describe in detail a computationally tractable posterior inference procedure if the prior on $(\beta_0, \ldots, \beta_k)$ is diffuse.

(c) Suppose that instead of working out the posterior distribution, you calculate the ML estimate and its associated standard errors. What can you say about the relationship between the two approaches in this case?

(d) Explain how to modify your procedure in part (b) if the prior has each component β_j independent with

$$p(\beta_j) \sim N(0, \sigma_j^2),$$

where

$$\sigma_j^2 = \frac{D}{j+1},$$

where D is a fixed constant. Why might you prefer a prior like this instead of the flat prior?

Note on Posterior Analysis of GCR Model:

Recall that in the classical regression model,

$$Y|X \sim N(X\beta, \Omega),$$

if the prior distribution for β is $N(a, H_1^{-1})$, then

$$\beta|Y, X, \Omega, \gamma \sim N(\beta^*, H_2^{-1}),$$

where

$$H = X'\Omega^{-1}X, \qquad H_2 = H + H_1,$$

$$\hat{\beta} = (X'\Omega^{-1}X)^{-1}X'\Omega^{-1}Y, \qquad \beta^* = H_2^{-1}(H\hat{\beta} + H_1 a).$$