Spring 2003 UCLA Department of Economics
Written Qualifying Examination in ECONOMETRICS

Instructions:
Answer **ALL questions** in Parts I, II, and III
Use a separate answer book for each Part.
You have four hours to complete the exam.
Calculators and other electronic devices are not allowed.

1 Part I

Question 1:

Suppose that we observe (x_i, y_i) $i = 1, \ldots, n$. They are assumed to be i.i.d., and they are assumed to satisfy the moment restriction

$$E[x_i] = E[y_i]$$

Let μ denote the common mean $E[x_i] = E[y_i]$.

1. Propose an efficient estimator of μ.
2. How would you test the restriction that $E[x_i] = E[y_i]$?

Question 2:

Suppose that you have a model

$$y_i = x_i \cdot \beta + \varepsilon_i$$

with the restriction

$$E[x_i \varepsilon_i] = 0$$

and

$$E[z_i \varepsilon_i] = 0$$

for some z_i. We would like to compare the asymptotic variance of $\sqrt{n}(b_{OLS} - \beta)$ with that of $\sqrt{n}(b_{GMM} - \beta)$, where b_{GMM} is the GMM estimator based on the two moment restrictions $E[x_i \varepsilon_i] = E[z_i \varepsilon_i] = 0$. Derive the asymptotic variances under the assumption

$$\begin{aligned} E[\varepsilon_i | x_i, z_i] &= 0 \\ E\left[\varepsilon_i^2 \middle| x_i, z_i\right] &= \sigma^2 \end{aligned}$$

Question 3:

Suppose that you have a model

$$\begin{aligned} y_i &= x_i \cdot \beta + \varepsilon_i \\ x_i &= z_i \cdot \pi + v_i \end{aligned}$$

such that z_i is independent of (ε_i, v_i). We assume that $E[\varepsilon_i] = 0$ and $E[v_i] = 0$. We further assume that $E[\varepsilon_i \cdot v_i] = 0$. Finally, we assume that the vector (y_i, x_i, z_i) $i = 1, \ldots, n$ is assumed to be i.i.d. (y_i, x_i, z_i are all scalars.)

1. Show that the OLS $\widehat{\beta}$ of y_i on x_i is consistent for β. What is the asymptotic distribution of $\sqrt{n}\left(\widehat{\beta}-\beta\right)$?

2. Let $\widehat{\gamma}$ denote the IV of y_i on x_i using z_i as instruments. What is the asymptotic distribution of $\sqrt{n}\left(\widehat{\gamma}-\beta\right)$? Adopt the simplifying notation $\Lambda \equiv E\left[z_i^2\right]$.

3. What is the asymptotic distribution of the vector

$$\begin{pmatrix} \sqrt{n}\left(\widehat{\beta}-\beta\right) \\ \sqrt{n}\left(\widehat{\gamma}-\beta\right) \end{pmatrix}$$

4. What is the asymptotic distribution of $\sqrt{n}\left(\widehat{\gamma}-\widehat{\beta}\right)$? How is the asymptotic variance here related to the asymptotic variances you derived before?

2 Part II

Question 1:

Discuss the asymptotic efficiency of the fixed effects estimator of β for the linear static panel data model:

$$y_{it} = x_{it}\beta + \alpha_i + \varepsilon_{it} \qquad\qquad i = 1,...,n; t = 1,...,T$$

where n is large relative to T. Prove any claims and state the assumptions made.

Question 2:

1. Write down a panel data Type I Tobit model with additive individual effects.

2. Write the log-likelihood of a random effects specification of the model that assumes a normal distribution for the composite error term. Make sure to allow the time-varying error component to be possibly heteroskedastic over time.

3. Describe Heckman's two-step method for estimating the model of part (2).

4. Describe a method for identifying a fixed effects specification of the model that does not require the parametrization of the distribution of the model's unobservables. Make sure to state the important assumptions that allow identification and to provide a consistent estimator for the model's parameters of interest.

5. What are the advantages and disadvantages of the approaches in (2) and (4)?

3 Part III

Question 1:

Consider the two moment functions given by $\varphi_1(W_i,\theta_1)$ and $\varphi_2(W_i,\theta_1,\theta_2)$. Suppose that, when evaluated at the true population parameter vectors, θ_{01} and θ_{02}, respectively, we have

$$\begin{aligned} E\left[\varphi_1(W_i,\theta_{01})\right] &= 0 \quad \text{and} \\ E\left[\varphi_2(W_i,\theta_{01},\theta_{02})\right] &= 0, \end{aligned}$$

where W_i, $i = 1, ..., n$, represent cross-section independent and identically distributed (iid) data.

Let $\varphi_1(\cdot)$ and $\varphi_2(\cdot)$ be $M_1 \times 1$ and $M_2 \times 1$ vector-valued functions, respectively, and correspondingly let θ_{01} and θ_{02} be d_1 and d_2 vectors of parameters, with $M_1 > d_1$ and $M_2 = d_2$.

1. For this part consider only $\varphi_1(\cdot)$. Suggest an efficient Generalized Method of Moments (GMM) estimator for θ_{01}.

2. Show that the estimator suggested in (1) is consistent and has an asymptotic normal distribution. In doing so explicitly state all the assumptions you make.

3. For this part assume that you already obtained an estimator for θ_{01}, say $\widehat{\theta}_{n1}$. A way to obtain an estimator for θ_{02}, say $\widehat{\theta}_{n2}$, is by (approximately) solving

$$\frac{1}{n}\sum_{i=1}^{n}\varphi_2(W_i, \widehat{\theta}_{n1}, \widehat{\theta}_{n2}) = 0,$$

   where $\widehat{\theta}_{n1}$ is a plug-in estimator that was obtained in an earlier stage (say in (1)). Provide the asymptotic properties of the estimator $\widehat{\theta}_{n2}$ for θ_{02}. Justify each and every step in your answer.

4. Suppose now that parameter vector θ_{01} is known, so that in order to obtain an estimator for θ_{02} we need to solve

$$\frac{1}{n}\sum_{i=1}^{n}\varphi_2(W_i, \theta_{01}, \widehat{\theta}_{n2}) = 0.$$

   How are the properties of the estimator obtained here different from those of the estimator obtained in (3)? Justify your answer.

5. For this part assume that $M_1 = d_1$. One suggested that instead of following the two-step procedure in (1)-(3) one can obtain estimators for θ_{01} and θ_{02} by simply solving

$$\frac{1}{n}\sum_{i=1}^{n}\varphi(W_i, \widehat{\theta}_{n1}, \widehat{\theta}_{n2}) = 0,$$

   for $\widehat{\theta}_{n1}$ and $\widehat{\theta}_{n2}$ simultaneously. Compare the properties of the estimators for θ_{01} and θ_{02} suggested here with those from the two-step procedure described above.

Question 2:

Consider the binary choice model given by

$$y_i = \begin{cases} 1 & \text{if } y_i^* > 0, \\ 0 & \text{otherwise,} \end{cases}$$

where

$$y_i^* = x_i'\gamma_0 + u_i$$

and

$$u_i | x_i \sim N\left(0, \sigma_u^2\right),$$

for $i = 1, ...n$.

1. Which of the model's parameters are identified? Justify your answer.

2. Write the likelihood function, say $L(\theta)$, where θ are all the model's parameters that are identified. Show that the Maximum Likelihood estimator (MLE), say $\widehat{\theta}_n$, satisfies

$$\sqrt{n}\left(\widehat{\theta}_n - \theta_0\right) \xrightarrow{D} N\left(0, I^{-1}(\theta_0)\right),$$

where θ_0 is the population's true parameter vector and $I(\theta_0)$ denotes the Fisher's information matrix, evaluated at θ_0.

3. For this part assume that θ_0 is a K-dimensional parameters vector, with $K > 10$. Suppose the we are interested in jointly testing the hypotheses that $\theta_{02} = \theta_{03}$, $\theta_{04} = \theta_{05}$, and $\sum_{k=2}^{K} \theta_{k0} = 1$. Provide details about how to use the Wald and Likelihood ratio (LR) tests for testing these hypotheses based on the first-order asymptotic results established in (1).

4. Suppose now that you are interested in testing the same hypotheses as in (3), with the Wald and LR tests, using the bootstrap method. Provide a detailed description of your proposed procedure.

5. Propose a procedure for computing an equal-tailed confidence interval for a known scalar function of the parameter vector θ_0, say $h(\theta_0)$.

6. Suppose now that it is too difficult to directly compute the probability

$$\Pr\left(y_i | x_i; \theta\right)$$

that appears in the likelihood function. Suggest a simulated method for obtaining an estimator for θ_0, say $\widehat{\theta}_n^S$.

7. How do the estimators, $\widehat{\theta}_n^S$ from (6) and $\widehat{\theta}_n$ from (1), differ from each other, if at all? Justify your answer.