UCLA Department of Economics

Second-Year Field Examination in ECONOMETRICS

Spring 2005

This is a 4 hour closed book/closed notes exam.

There are FOUR parts in the exam.
Answer ALL questions of ONLY THREE parts of your choice.
Use a separate bluebook for each part.

GOOD LUCK!

PART I (Buchinsky)

Please answer both Questions.

1. Consider the non-linear cross-sectional model

$$\begin{aligned} y_i &= h(x_i;\theta_0) + \varepsilon_i, \quad \text{with} \\ \varepsilon_i &= g(x_i)\, u_i, \quad \text{where} \\ u_i|x_i &\sim \text{i.i.d.}\left(0, \sigma_u^2\right), \end{aligned} \tag{1}$$

for $i = 1, ..., n$, where x_i is a vector of regressors, θ_0 is a vector of parameters, both are $K \times 1$ vectors, and $g(\cdot)$ and $h(\cdot)$ are known functions.

(a) Compute $E[y_i|x_i]$.

(b) Consider the estimator for θ_0, say $\widehat{\theta}_n^{LS}$ obtained by minimizing

$$Q_n(\theta) = \frac{1}{n}\sum_{i=1}^{n}(y_i - h(x_i;\theta))^2$$

with respect to θ. Show that the estimator is consistent and asymptotically normal. Make sure you explain all the key assumptions needed for establishing consistency and asymptotic normality.

(c) Provide the covariance matrix for the asymptotic distribution obtained in (b), and suggest a consistent estimator for that asymptotic covariance matrix.

(d) Now consider the estimator obtained by minimizing

$$Q_n^*(\theta) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{y_i - h(x_i;\theta)}{g(x_i)}\right)^2.$$

Show that the estimator, say $\widehat{\theta}_n^*$, is consistent and asymptotically normal, and provide the asymptotic covariance matrix for the asymptotic normal distribution.

(e) Which of the estimators would you prefer, $\widehat{\theta}_n^{LS}$ from (b), or $\widehat{\theta}_n^*$ from (d). Justify your answer briefly.

(f) Suppose now that the error term u_i has a normal distribution. That is,

$$u_i|x_i \sim \text{i.i.d. N}\left(0, \sigma_u^2\right).$$

Can you propose an estimator that will be better than the two previously considered? Please give as detailed an answer as possible.

2. Consider the linear regression model

$$y_i = x_i'\beta_0 + \varepsilon_i,$$

for $i = 1, ..., n$, where β_0 is a $K \times 1$ vector of parameters and x_i is a vector of regressors, some of which are endogenous, that is,

$$E[\varepsilon_i|x_i] \neq 0.$$

Also suppose that we have an $M \times 1$ vector of regressors z_i, with $M > K$, such that

$$E[\varepsilon_i|z_i] = 0.$$

(a) Suggest a consistent estimator for β_0. Show that the estimator is in fact consistent.

(b) Suppose that the estimator also has an asymptotic normal distribution. However, we would like to use the bootstrap method to compute the standard error, confidence interval, etc. Is the bootstrap method valid in the current case? Justify your answer.

(c) Suppose the we want to test the hypothesis H$_0$: $\sum_{k=1}^{K} \beta_{0k} = 1$, against the alternative hypothesis H$_1$: $\sum_{k=1}^{K} \beta_{0k} > 1$. Describe in details how to use the bootstrap method for testing this hypothesis. (Hints: the bootstrap method should be used to construct the rejection region.)

(d) How would you use the bootstrap method to construct a two-side equal-tail $(1-\alpha)*100$ confidence interval for the true parameter $\gamma_0 = \sum_{k=1}^{K} \beta_{0k}$.

(e) Suppose that you want to use the bootstrap repetitions in (d) to construct a standard error estimate, say $\widehat{se}_\gamma$ for the parameter estimate for γ_0, say $\widehat{\gamma}_n$. How would you do that?

(f) It was claimed that an alternative symmetric confidence interval for γ_0 can also be provided by

$$CI_A(\gamma_0) = \left[\widehat{\gamma}_n - \frac{\widehat{se}_\gamma}{\sqrt{n}} Z_{1-\alpha/2},\ \widehat{\gamma}_n - \frac{\widehat{se}_\gamma}{\sqrt{n}} Z_{1-\alpha/2}\right].$$

Would $CI_A(\gamma_0)$ be a valid confidence interval for γ_0? Justify your answer.

(g) Regardless of your answer in (f), assume now that $CI_A(\gamma_0)$ is a valid confidence interval for γ_0. Which of the two confidence intervals would you prefer - the one constructed in (d) or $CI_A(\gamma_0)$ from (f)? Justify your answer in detail.

PART II—(Kyriazidou)

Please answer both questions.

Question 1: RIGHT OR WRONG; PROVE YOUR CLAIM

(a) The First Difference estimator is consistent in the static linear panel data model

$$y_{it} = x_{it}\beta + \alpha_i + \varepsilon_{it}$$

provided that $E\left(x_{it}\varepsilon_{it}\right) = 0$ for all t.

(b) The GLS estimator is BLUE in the static linear panel data model

$$y_{it} = x_{it}\beta + \alpha_i + \varepsilon_{it}$$

provided that $E\left(x_{it}\varepsilon_{is}\right) = 0$ and $E\left(\alpha_i x_{it}\right) = 0$ for all t, s.

(c) The ML estimator of $\left(\beta, \sigma^2\right)$ is consistent in the normal static linear fixed effects model

$$\begin{aligned} y_{it} &= x_{it}\beta + \alpha_i + \varepsilon_{it} \\ \varepsilon_{it}|x_i, \alpha_i &\sim iidN\left(0, \sigma^2\right) \end{aligned}$$

when the α_i's are estimated along with β, σ^2.

Question 2:

(a) Describe and justify the kernel estimator of the univariate regression function $E\left(Y_i|X_i = x\right)$ when i.i.d data on $\left(Y_i, X_i\right)$ are available.

(b) Show that it is consistent stating all necessary assumptions.

(c) State its asymptotic distribution and provide an estimator for its asymptotic variance.

PART III (Giacomini)

Please answer two out of the following three questions.

1. In the out-of-sample forecast evaluation literature, one often finds the regression

$$e_{t,\tau} = \alpha + u_{t+\tau}, \tag{2}$$

where $e_{t,\tau}$ are out-of-sample $\tau-$steps-ahead forecast errors for some model.

   (a) What estimator of the variance of $\hat{\alpha}$ would you use in a t-test of the hypothesis H_0 : $\alpha = 0$? Why?

   (b) Explain why a t-test of $H_0 : \alpha = 0$ corresponds to testing an implication of forecast optimality (unbiasedness) only when the loss function is quadratic (i.e., $L(e_{t,\tau}) = e^2_{t,\tau}$).

   (c) Consider a linex loss: $L(e_{t,\tau}) = \exp(e_{t,\tau}) - e_{t,\tau} - 1$. Find the expectation of the *optimal* forecast errors under the assumption that they are normally distributed. (Hint: if $Y \sim N(\mu, \sigma^2)$, then $E\exp(Y) = \exp(\mu + \sigma^2/2)$).

   (d) Explain how you would modify equation (2) to test for unbiasedness if you have a linex loss function.

2. For a sample of size T, write the sample loglikelihood for the following model:

$$\begin{aligned} Y_t &= \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t; \\ \varepsilon_t &= \sigma_t z_t, \end{aligned}$$

where z_t is iid with mean zero and variance 1 and is distributed as a generalized error distribution (GED) variable with parameter ν, which has the following pdf:

$$\begin{aligned} f_Z(z) &= \frac{\nu \exp\left(-\frac{1}{2}\left|\frac{z}{\lambda}\right|^{\nu}\right)}{\lambda 2^{(1+\frac{1}{\nu})}\Gamma(\frac{1}{\nu})}, \text{ where} \\ \lambda &= \left(\frac{2^{-\frac{2}{\nu}}\Gamma(\frac{1}{\nu})}{\Gamma(\frac{3}{\nu})}\right)^{1/2}. \end{aligned}$$

3. Let x_t = log of output; m_t = log of money supply; u_{st} = iid supply shock; u_{mt} = iid money shock such that $Eu_{st}u_{m\tau} = 0$ for all t, τ. The interpretation of the shocks is such that $u_{mt} > 0$ means an increase in money supply and $u_{st} > 0$ means bad news. Consider the SVAR(1)

$$\begin{aligned} A_0 Y_t &= A_1 Y_{t-1} + u_t, \; Eu_t u_t' = \Omega_u \\ Y_t &= \begin{pmatrix} \Delta x_t \\ \Delta m_t \end{pmatrix}; \; u_t = \begin{pmatrix} u_{mt} \\ u_{st} \end{pmatrix} \end{aligned} \tag{3}$$

and the corresponding reduced-form VAR(1)

$$Y_t = \Phi Y_{t-1} + \varepsilon_t, \; E\varepsilon_t\varepsilon_t' = \Omega_\varepsilon. \tag{4}$$

Suppose that the estimation of (4) yields reduced-form parameters $\Phi = \begin{pmatrix} .4 & -.4 \\ .1 & .9 \end{pmatrix}$, $\Omega_\varepsilon = \begin{pmatrix} 16 & 4 \\ 4 & 26 \end{pmatrix}$.

(a) Under the assumption that money does not contemporaneously affect output, derive the structural parameters in (3).

(b) Using the identifying assumption in point a, find the instantaneous response of output and money to a one standard deviation shock to both u_{mt} and u_{st}. Are the signs reasonable? (You need to compute actual numbers for this question)

(c) Using the identifying assumption in point a, how would you compute the impulse response function (IRF) of both output and money to a money shock? The IRF of output and money to a supply shock?

(d) Now suppose you make the assumption that money shocks have no *long-run* effect on the level of output. Explain how you would obtain IRFs under this alternative identifying restriction. (For this question, you don't need to compute actual numbers).

PART IV (Guggenberger)

Question 1) True/Questionable/False? (No points are given for just stating true/questionable or false. The explanation is what counts.)

(i) For the Geweke Porter–Hudak estimator $\widehat{d}$ of the long memory parameter d the number of frequencies m used in the OLS regression has to go to infinity with increasing sample size to assure that the variance of $\widehat{d}$ goes to 0.

(ii) If one regresses a random walk $y_{1,t}$ on its own lagged observation $y_{1,t-1}$, on an independent random walk $y_{2,t}$ and on its lagged observation $y_{2,t-1}$ i.e. $y_{1,t} = \widehat{\alpha}y_{1,t-1} + \widehat{\beta}y_{2,t} + \widehat{\gamma}y_{2,t-1}$ then the theory of spurious regression implies that the OLS estimator of β is inconsistent.

(iii) A necessary condition for the validity of the bootstrap is that the limit distribution $H(x,\theta) = \lim_{T\to\infty} P[Y_T(\widehat{\theta}_T - \theta) \leq x]$ in the parameter of interest θ does not vary discontinuously in θ. (Y_T is a sequence of normalizing coefficients.)

(iv) Valid inference with weak instruments is typically uninformative.

Question 2: The linear model is given by the structural and reduced form equations $y = Y\theta_0 + u$ and $Y = Z\Pi + V$ for $\theta_0 \in R^p$ and instruments $Z \in R^{n\times k}$.

i) Explain what it means that some components of θ_0 are weakly identified and describe the "local to zero" asymptotic framework.

ii) Describe in detail three different approaches to test $H_0 : \theta_1 = \theta_{10}$, where $\theta_0 = (\theta_{10}', \theta_{20}')'$, against the alternative $H_0 : \theta_1 \neq \theta_{10}$, when θ_{10} is potentially only weakly identified. What are the assumptions needed for valid inference for each of the three approaches? Describe the advantages and disadvantages of the approaches.

Question 3: i) Precisely state Donsker's theorem and the continuous mapping theorem with all the assumptions needed. What do we achieve by a Beveridge-Nelson decomposition?

ii) The Phillips and Perron statistic for a unit root test in the model

$$y_t = \alpha + \rho y_{t-1} + u_t,$$

with possibly serially correlated and heteroskedastic u_t reads

$$T(\widehat{\rho}_T - 1) - (T^2\widehat{\sigma}^2_{\widehat{\rho}_T}/s_T^2)(\lambda^2 - s_T^2)/2, \tag{5}$$

where $\widehat{\rho}_T$ is the OLS estimator of ρ, $\widehat{\sigma}_{\widehat{\rho}_T}$ the OLS standard error for $\widehat{\rho}_T$, s_T^2 the OLS estimate of the variance of u_t and λ^2 the long run variance of u_t. To implement the test, we need to replace λ^2 by a HAC estimator which requires the choice of a bandwidth. To circumvent this complication, someone suggests to use the statistic

$$T(\widehat{\rho}_T - 1) + T^2\widehat{\sigma}^2_{\widehat{\rho}_T}/2, \tag{6}$$

instead of (5). Derive its asymptotic distribution and show that it does not depend on nuisance parameters.

iii) Under the alternative $\rho < 1$ we have $T(\widehat{\rho}_T - 1) \to -\infty$ and the null $\rho = 1$ is rejected if the test statistics (5) and (6) are smaller than an appropriate threshold. Comparing the statistics (5) and (6) which test is likely to have better power properties? Explain.