FIELD EXAM

Econometrics

Thursday, June 25, 2009

TIME: 9am – 1pm

Instruction: Skip one section, and solve all remaining sections. Each section has its own set of rules.

Section 1

Please answer all questions:

1. Consider the panel data model given by:

$$y_{it} = \alpha_i + x_{it}'\beta_0 + u_{it},$$

for $i = 1, ..., N$, and $t = 1, ..., T$, where β_0 is a $K \times 1$ vector of unknown parameters, $x_{it}' = (x_{1it}, ... x_{Kit})$, α_i is fixed parameter that may be correlated with x_{it}, and $E[u_{it}|x_{it}] = 0$.

   (a) Provide the *within* and *between* estimators for β_0, say $\widehat{\beta}_W$ and $\widehat{\beta}_B$, respectively.

   (b) Suppose now that x_{1it} is time invariant, that is, $x_{1it} = x_{1i}$, for all $t = 1, ..., T$. Discuss which of the model's parameters can be identified. Justify your answer.

   (c) Now consider the correlated random effect model, where

$$\alpha_i = x_i'\delta_0 + v_i,$$
$$x_i' = (x_{i1}', ..., x_{iT}'),$$

   where $E[v_i|x_{it}] = 0$. The model can be written in matrix form as

$$y_i = \Pi x_i + \varepsilon_i,$$

   where $y_i' = (y_{i1}, ..., y_{iT})$. Provide Π and explain the structure of ε_i. In particular, provide the variance-covariance matrix of ε_i, say Σ_ε. Suggest a consistent estimator for Σ_ε, say $\widehat{\Sigma}_\varepsilon$.

   (d) Provide details as to how one should go about estimating Π. In particular provide the efficient estimator for Π.

2. Consider the non-linear regression model

$$y_i = g(x_i'\beta_0) + \varepsilon_i,$$

for $i = 1, ..., N$, where β_0 is a $K \times 1$ vector of parameters and x_i is a vector of regressors, with $E[\varepsilon_i|x_i] = 0$

   (a) Suggest a consistent estimator for β_0. Show that the estimator is in fact consistent.

(b) Suppose that the estimator also has an asymptotic normal distribution. But, we would like to use the bootstrap method to compute the standard error, confidence interval, etc. Is the bootstrap method valid in the current case? Justify your answer.

(c) Suppose the we want to test the hypothesis H$_0$: $\sum_{k=1}^{K} \beta_{0k} = 1$, against the alternative hypothesis H$_1$: $\sum_{k=1}^{K} \beta_{0k} > 1$. Describe in details how to use the bootstrap method for testing this hypothesis. (Hints: the bootstrap method should be used to construct the rejection region.)

(d) How would you use the bootstrap method to construct a two-side equal-tail $(1-2\alpha)*$ 100 confidence interval for the true parameter $\gamma_0 = \sum_{k=1}^{K} \beta_{0k}$. (You need not provide the exact formulas. Just describe the procedure that one should follow.)

(e) Suppose that you want to use the bootstrap repetitions in (d) to construct a standard error estimate, say $\widehat{se}_\gamma$ for the parameter estimate for γ_0, say $\widehat{\gamma}_n$. How would you do that?

(f) It was claimed that an alternative symmetric confidence interval for γ_0 can also be provided by

$$CI_A(\gamma_0) = \left[\widehat{\gamma}_n - \frac{\widehat{se}_\gamma}{\sqrt{n}} Z_{1-\alpha/2},\ \widehat{\gamma}_n + \frac{\widehat{se}_\gamma}{\sqrt{n}} Z_{1-\alpha/2}\right].$$

Would $CI_A(\gamma_0)$ be a valid confidence interval for γ_0? Justify your answer.

Section 2

Please answer all questions:

1. Precisely state Donsker's theorem and the continuous mapping theorem with all the assumptions needed. What do we achieve by a Beveridge-Nelson decomposition?

2. The Phillips and Perron test statistic for a unit root test

$$H_0 : \rho = 1 \text{ versus } H_1 : \rho < 1 \tag{1}$$

   in the model

$$y_t = \alpha + \rho y_{t-1} + u_t$$

   with possibly serially correlated and heteroskedastic u_t reads

$$T(\widehat{\rho}_T - 1) - (T^2 \widehat{\sigma}^2_{\widehat{\rho}_T} / s^2_T)(\lambda^2 - s^2_T)/2, \tag{2}$$

   where T is the sample size, $\widehat{\rho}_T$ the OLS estimator of ρ, $\widehat{\sigma}_{\widehat{\rho}_T}$ the OLS standard error for $\widehat{\rho}_T$, s^2_T the OLS estimate of the variance of u_t, and λ^2 a HAC estimator of the long run variance of u_t. The latter estimator requires the choice of a bandwidth. Derive the limit distribution of the statistic (show all your work!) under the null hypothesis and explain how to implement the test. Clearly state any assumptions that you need.

3. To circumvent the need for a HAC estimator and a bandwidth choice, someone suggests to use the statistic

$$T(\widehat{\rho}_T - 1) + T^2 \widehat{\sigma}^2_{\widehat{\rho}_T}/2, \tag{3}$$

   instead of (2) to test (1). Derive its asymptotic distribution (note that this is a subresult of part 2)) and show that it does not depend on nuisance parameters. How could this statistic be used for a unit root test that asymptotically controls the null rejection probability?

4. Compare the tests based on (2) and (3) in terms of their power properties against fixed alternatives $\rho < 1$.

5. Explain in what sense it is more complicated to construct a confidence interval (CI) for ρ than to test (1). What is the problem of the standard (two–sided symmetric) CI for ρ based on inverting a t test? Explain. Discuss how to implement the CI suggested by Andrews (1993). Explain how to obtain a two–sided symmetric subsampling CI for ρ based on inverting a t test (based on the OLS estimator).

Section 3

Please answer all questions:

1. Consider the following model

$$y_{it} = \alpha_i \beta + \varepsilon_{it}$$

$$\alpha_i = x_{it} + v_{it}$$

where x_{it} is independent of $(\varepsilon_{it}, v_{it})$. The vector $(\varepsilon_{it}, v_{it})$ has mean equal to zero. The vector $(\varepsilon_{it}, v_{it})$ is assumed to have mean zero. We will assume that $t = 1, \ldots, T$ and T is **fixed**. We will adopt the asymptotic approximation where $i = 1, \ldots, n$ and $n \to \infty$. For simplicity, assume that β is a scalar. Prove that the 2SLS for β is numerically equivalent to

$$\frac{\sum_{i,t} \overline{x}_i y_{it}}{\sum_{i,t} \overline{x}_i x_{it}} = \frac{\sum_{i,t} \overline{x}_i \overline{y}_i}{\sum_{i,t} \overline{x}_i^2}$$

where $\overline{x}_i = \frac{1}{T} \sum_{t=1}^{T} x_{it}$. Is the 2SLS consistent for β? If not, propose a consistent estimator. Prove why your estimator is consistent.

2. Consider the panel model

$$y_{it} = \alpha_i + x_{it}\beta + \varepsilon_{it}$$

where $t = 1, 2$ and $i = 1, \ldots, n$. For simplicity, assume that β is a scalar. We will adopt the asymptotic approximation where $n \to \infty$. Suppose as Mundlak that

$$\alpha_i = \overline{x}_i c + v_i$$

where $\overline{x}_i = \frac{1}{2} \sum_{t=1}^{2} x_{it}$, and $(v_i, \varepsilon_{i1}, \varepsilon_{i2})$ is independent of (x_{i1}, x_{i2}). Assume that you only observe (y_{i1}, x_{i1}, x_{i2}), i.e., we do NOT observe y_{i2}. Prove that you can consistently estimate β. Now suppose as Chamberlain that

$$\alpha_i = x_{i1}\pi_1 + x_{i2}\pi_2 + v_i$$

and $(v_i, \varepsilon_{i1}, \varepsilon_{i2})$ is independent of (x_{i1}, x_{i2}). Assume as before that you only observe (y_{i1}, x_{i1}, x_{i2}). Can you still estimate β consistently?

3. Consider a model of treatment effects discussed in class, where we assume

$$D_i \perp\!\!\!\perp (Y_{1i}, Y_{0i}) | \, X_i$$

Let $Y_i = D_i Y_{1i} + (1 - D_i) Y_{0i}$. Also, let $p(X_i) \equiv \Pr[D_i = 1 | \, X_i]$ denote the propensity score. You only observe (Y_i, X_i, D_i). Assuming that X_i has a binary distribution, suggest a consistent estimator for $E[Y_{1i} - Y_{0i}]$. Argue why your estimator is consistent.

Section 4

Please answer all questions:

1. This question consists of two parts.

Part 1: Consider a binary response model of the type

$$Y^* = m(X) - \varepsilon$$

$$Y = 1 \quad if \quad Y^* \geq \alpha$$
$$Y = 0 \quad otherwise$$

where $Y \in \{0, 1\}$ and $X \in R^K$ are observable, and ε and Y^* are scalar unobservables. The scalar α is strictly positive and unknown. Suppose that ε is distributed independently of X. Denote the distribution of ε by F_ε and the support of X by D.

(a) Derive an expression for the probability of Y given $X = x$, when $x \in D$, in terms of m and F_ε.

(b) Analyze the identification of α and m when $D = \{x^1, ..., x^N\}$ is a set containing only a finite number of points and F_ε is a known function. (In other words, determine first what can and cannot be identified about α and m, and second, provide a set of minimal conditions under which α and m are identified.)

(c) Analyze the identification of α, m and F_ε when α, m and F_ε are unknown, $D = R^K$, and m is homogeneous of degree one, i.e., for all $\lambda > 0$, $m(\lambda x) = \lambda m(x)$.

(d) Suppose now that $\alpha = 0$, $m(X_1, ..., X_K) = \sum_{k=1}^{K} m_k(X_k)$, and F_ε and the m_k functions are unknown. Provide a minimal set of conditions under which the functions m_k and the distribution F_ε are identified when the support of X is R^K.

Part 2: Consider now the following binary response model

$$Y^* = Z + \varepsilon + m(X, \eta)$$

$$Y = 1 \quad if \quad Y^* \geq \alpha$$
$$Y = 0 \quad otherwise$$

where $Y \in \{0,1\}$, $X \in R^K$ and $Z \in R$ are observable, ε, η and Y^* are scalar unobservables, and for all values z, x, ε, η,

$$f_{Z,X,\varepsilon,\eta}(z,x,\varepsilon,\eta) = f_Z(z) f_X(x) f_\varepsilon(\varepsilon) f_\eta(\eta)$$

where f_w denote the density of w. Analyze the identification of α, the distributions of ε and of η and of the function m, assuming that the support of Z is R, the support of X is R^K, and that there exists values $\widetilde{x}$ and $\overline{x}$ of X such that for all η

$$m_k\left(\widetilde{x}, \eta\right) = 0, \text{ and}$$
$$m_k\left(\overline{x}, \eta\right) = \eta.$$

2. Suppose that you wanted to estimate the unknown, concave, monotone increasing and continuous function m in the model

$$Y = m\left(X\right) + \varepsilon$$

where for all x, $E\left(\varepsilon | X = x\right) = 0$.

(a) Describe two consistent estimators for m, one using all the shape restrictions on m and one only using the continuity of m.

(b) Justify your answer to (a).

(c) Suggest a procedure to estimate m consistently when $E\left(\varepsilon | X = x\right) \neq 0$ but for some observable Z, and all x, z, $E\left(\varepsilon | X = x, Z = z\right) = 0$.